Propositional Logic

Propositional logic deals with statements (propositions) that are either true or false.

For example, these are all valid propositions:

A square has 4 sides
George Washington was President of France
There is a cherry tree in the garden of Holyrood House

They are valid propositions because each one of them can be shown to be either true or false:

The first one is definitely true.
The second is definitely false.
I don't know whether the first is true or false, but it would be possible to go to Holyrood House and find out.

Propositional Logic

Propositional logic deals with statements (propositions) that are either true or false.

Why are these 3 statements NOT valid propositions in propositional logic?

1. Everything in this topic is untrue
2. Spinach tastes better than cabbage
3. They are intelligent

1. This statement cannot be either true or false - if it is true, then it must be untrue - if it is false, then it must be true!
2. This statement is a matter of opinion - some people will consider it true, some will consider it false
3. The truth of this statement cannot be determined, because we do not know who "they" are, and there is no agreed definition of "intelligent".

Propositional Logic

Propositional logic is very useful, and forms the basis for almost all other types of logic.
However, it has one serious weakness - it only deals with whole propositions or sentences. The following example illustrates this:

Let P represent "All teachers are human"
Let Q represent "David is a teacher"
Let R represent "David is human"

In propositional logic, we can write that $P \wedge Q \rightarrow R$ is true (if all teachers are human, and David is a teacher, then David is human).

What about Anne? or Seamus? or Khalid?

In propositional logic, we would need to write a new rule for each person. We have no way of making a proposition general.

Propositional Logic

Let S represent "It is snowing"

Let F represent "It is below freezing"

Let G represent "I go out to play"

How would you represent the following statements:

1. It is not snowing — $\neg S$
2. It is snowing and it is below freezing — $S \wedge F$
3. If it is below freezing I do not go out to play — $F \rightarrow \neg G$
4. I go out to play if it is snowing — $S \rightarrow G$
5. If it is snowing and it is not below freezing, I go out to play — $S \wedge \neg F \rightarrow G$
6. If it is snowing or it is not below freezing, I go out to play — $S \vee \neg F \rightarrow G$

Propositional Logic

Consider:

Let P represent "All men are mortal"
Let Q represent "Napoleon is a man"
Let R represent Napoleon is mortal"

In propositional logic, we can write:

$P \wedge Q \rightarrow R$ (if all men are mortal, and Napoleon is a man, then Napoleon is mortal).

Consider the rule:
Let S represent "Socrates is a man"
Can we conclude that Socrates is mortal?
Using human logic, the answer would be yes, but with propositional logic another individual rule would have to be written:
Let T represent "Socrates is mortal" $P \wedge S \rightarrow T$

We cannot write a general rule applying to all mortal men in propositional logic. For each person who exists a new rule has to be written.

Predicate Logic

Propositional logic is very useful, and forms the basis for almost all other types of logic. However, it has one serious weakness - it only deals with whole propositions or sentences. The following example illustrates this:

Let P represent "All teachers are human"
Let Q represent "David is a teacher"
Let R represent "David is human"

It would be more useful, if we could write a rule to represent the statement "if all teachers are human, and X is a teacher, then X is a human", where X can represent anyone at all.
Propositional logic does not allow this, but predicate (1st order) logic does.

This uses a new operator, $\forall$ Meaning "for all".

Now we can write:

$$\forall (x): \text{teacher}(x) \rightarrow \text{human}(x)$$

For every x, if x is a teacher, then x is human.

Predicate Logic

Statement

Bouncer is a dog
Bouncer is a dog with a long tail
If Bouncer is a dog, he can bark.
All dogs can bark
No humans live on Mars
Even number are whole numbers divisible by 2
Every boy likes football
Two people are brothers if they have the same father

Logic Operators

∀	for all
∧	and
∨	or
¬	not
→	if/then

Dog(Bouncer)

Dog(Bouncer) ∧ Long_tailed(Bouncer)

Dog(Bouncer) → Can_bark(Bouncer)

∀(x): Dog(x) → Can_bark(x)

∀(x): Human(x) → ¬ Lives_Mars(x)

∀(x): Whole_number(x) ∧ Divides_by_2(x) → Even_number(x)

∀(x): Boy(x) → Likes_football(x)

∀(x,y,z): boy(x) ∧ boy(y) ∧ father(z,x) ∧ father(z,y) → brother(x,y)

Predicate Logic

City(Dundee) ¬ has_castle(Dundee)

Statement

Dundee is a city with no castle
If Dundee is a city, it has many inhabitants.
All cities have many inhabitants.
There are no cities on Mars.
A bungalow is a house with one floor.
Every woman likes flowers
Two people are cousins if they have the same grandparent

City(Dundee) ∧ ¬ has_castle(Dundee)

City(Dundee) → many_inhabitants(Dundee)

∀ (x): City(x) → many_inhabitants(x)

∀ (x): City(x) → ¬ On_mars(x)

∀ (x): House(x) ∧ One_floor(x) → Bungalow(x)

∀(x): Woman(x) → Likes_flowers(x)

∀(x,y,z): grandparent(x,y) ∧ grandparent(x,z) ∧ ¬ Sibling(y,z) → Cousin(y,z)